

## Measurement of Strain using a Borehole Tensor Strain instrument.

At the surface of the earth, the vertical stress is zero, and so the equations for the plane stress case can be used to describe strains in the horizontal plane. In this case, at point P in the body, let the Cartesian components of horizontal displacement be  $u$  in the  $x$  (east) direction, and  $v$  in the  $y$  (north) direction, and the normal convention of compression negative. The horizontal strain tensor at this point has three independent components,  $\epsilon_{xx}$ ,  $\epsilon_{yy}$  and  $\epsilon_{xy}$ , such that

$$\epsilon_{xx} = \frac{\partial u}{\partial x}; \quad \epsilon_{yy} = \frac{\partial v}{\partial y}; \quad \epsilon_{xy} = \epsilon_{yx} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

Elongation at an angle  $\theta$  is a linear combination of these three components  $\epsilon_{xx}$ ,  $\epsilon_{yy}$  and  $\epsilon_{xy}$ . In principle three independent observations in the horizontal plane of elongation in a body should be sufficient to determine the strain tensor at that position. This is the primary motivation for the construction of a borehole strain measurement instrument with three independent measurement modules, in which the diameter is measured to a precision of 1 part in  $10^{10}$ .

Traditional engineering analysis has led to a formalism of quantities  $\epsilon_a$ ,  $\gamma_1$  and  $\gamma_2$  such that

$$\epsilon_a = \epsilon_{xx} + \epsilon_{yy}$$

$$\gamma_1 = \epsilon_{xx} - \epsilon_{yy}$$

$$\gamma_2 = 2\epsilon_{xy}$$

These also completely describe the strain state, but isolate the areal or compressional strain  $\epsilon_a$  from the shear strain in  $\gamma_1$  and  $\gamma_2$

$\gamma_1$  is a pure engineering shear which is maximum across planes oriented northwest or northeast (ie. at 45 degrees to the coordinate system).  $\gamma_2$  is a pure engineering shear with maximum shear across planes N-S and E-W.

After accounting for the effects of earth tides, ocean loading, local topography and geology, and calibration using earth tidal signals, strains can be determined to better than 1 part in  $10^9$  (1 Nanostrain), and with a long term stability of better than 100 Nanostrain per year.